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Numerical methods

MME308

Assignment 3

Grop.

Problem no: 1,6,7,11

fall 2014

Problem 1

Given

x	2	2.3	2.6	2.9	3.2
y	5.1	7.2	9.5	14.1	20

$$aN + b\sum_{i=1}^{n} xi = \sum_{i=1}^{n} yi$$

$$a\sum_{i=1}^{n} xi + b\sum_{i=1}^{n} xi2 = \sum_{i=1}^{n} xiyi$$

(1)

Required

Fit the following data to the:

- *A) least – square line $Y = a_0 + a_1 X$*
- *B) exponential function $Y = a_0 e^{a1X}$*

Solution :

i	X	Y	X2	X*Y
1	2	5.1	4.00	10.20
2	2.3	7.2	5.29	16.56
3	2.6	9.5	6.76	24.70
4	2.9	14.1	8.41	40.89
5	3.2	20	10.24	64.00
$\sum$	13.00	55.90	34.70	156.35

Now, eq (1) becomes .

$$5 A + 13 B = 55.9$$

(1)`

$$13 A + 34.7 B = 156.35$$

- *in this system is solving by using guess elimination*

$$\begin{pmatrix} 5 & 13 \\ 13 & 34.7 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 55.9 \\ 156.35 \end{pmatrix}$$

R1= R1/5

$$\begin{pmatrix} 1 & 2.6 \\ 13 & 34.7 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 11.18 \\ 156.35 \end{pmatrix}$$

R2= R2-13R1

$$\begin{pmatrix} 1 & 2.6 \\ 0 & 0.9 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 11.18 \\ 11.01 \end{pmatrix}$$

R2= R2/0.9

$$\begin{pmatrix} 1 & 2.6 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 11.18 \\ 12.233 \end{pmatrix}$$

- **Now, this augmented matrix represents the equivalent linear system.**

$$1 A + 2.6 B = 11.18 \qquad (2)$$

$$B = 12.233 \qquad (3)$$

- *Since* B = 12.233 *from the last equation ,substituting in the equation (2) by* B

$$A + 2.6B = 11.18$$

$$A + 2.6(12.2333) = 11.18$$

That is, $\qquad A = -20.6268$

Hence , the solution set consists of a_0 = - **20.6268** , a_1 = **12.2333** ,

Hence the fitting least square line

$$Y = a_1 * X + a_0$$

$$Y = 12.2333 * X - 20.6268$$

$$S_r = \sum_{i=1}^{n}(yi - f(xi))^2$$

i	X	y	Sr
1	2	5.1	1.5881
2	2.3	7.2	0.0960
3	2.6	9.5	2.8217
4	2.9	14.1	0.5622
5	3.2	20	2.1911
$\sum$	13	55.9	7.2590

$S_r = 7.259$

$$S_t = \sum_{i=1}^{n}(yi - y)^2$$

$X = \dfrac{13}{5} = 2.6$ $y = \dfrac{55.9}{5} = 11.18$

i	y	Sr
1	5.1	36.9664
2	7.2	15.8404
3	9.5	2.8224
4	14.1	8.5264
5	20	77.7924
$\sum$		141.9

$S_t = 141.9$

$$R = \sqrt{\frac{st - sr}{st}} = \sqrt{\frac{141.9 - 7.259}{141.9}} = 0.9741$$

$r^2 = 0.9488$, *so* 94.88 *of the original data are represented by our linear model.*

- Graph New the equation $y = a_0 + a_1 X$

y = 12.233x - 20.627
R² = 0.9489

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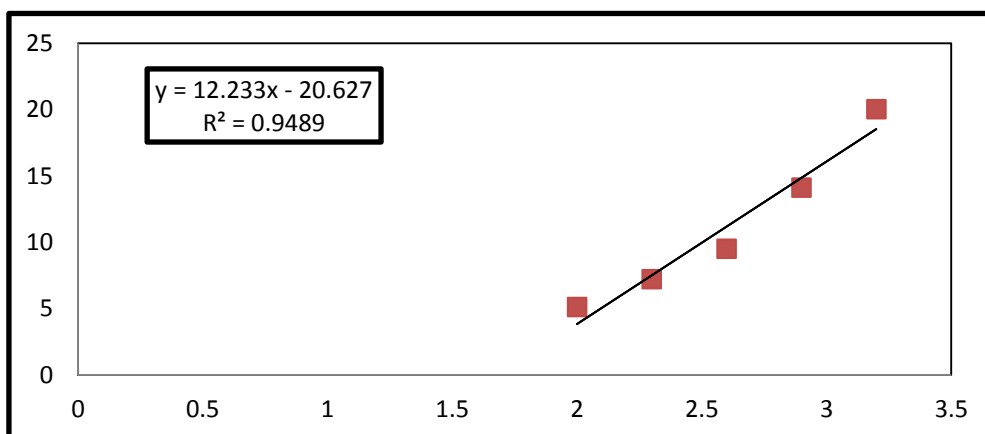
Problem 1 b

2- $Y = a_0 e^{a_1 X}$

i	X	y	Y=ln(y)	X*Y	X2
1	2	5.1	1.6292	3.2585	4
2	2.3	7.2	1.9741	4.5404	5.29
3	2.6	9.5	2.2513	5.8534	6.76
4	2.9	14.1	2.6462	7.6739	8.41
5	3.2	20	2.9957	9.5863	10.24
$\sum$	13		11.4965	30.9125	34.70

Now, eq (1) becomes .

$$5 A + 13 B = 11.4965$$

$$13 A + 34.7 B = 30.9125$$

(1)``

- By Using Guess elimination . in this system is solving

$$\begin{pmatrix} 5 & 13 \\ 13 & 34.7 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 11.4965 \\ 30.9125 \end{pmatrix}$$

R1= R1/5

$$\begin{pmatrix} 1 & 2.6 \\ 13 & 34.7 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 2.2993 \\ 30.9125 \end{pmatrix}$$

R2= R2-13R1

$$\begin{pmatrix} 1 & 2.6 \\ 0 & 0.9 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 2.2993 \\ 1.0216 \end{pmatrix}$$

R2= R2/0.9

$$\begin{pmatrix} 1 & 2.6 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 2.2993 \\ 1.1351 \end{pmatrix}$$

- Now, this augmented matrix represents the equivalent linear system.

$$1 A + 2.6 B = 2.2993 \ (2)$$

$$B = 1.1351 \ (3)$$

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- **Since B = 1.1351** from the last equation ,substituting in the equation (2) by **B**

$$A \ + \ 2.6B = 2.2993$$

$$A \ + \ 2.6(1.1351) = 2.2993$$

That is, $\qquad\qquad A = -0.6519$

but $A = ln(a_0)$, $a_0 = e^A$, $B = a_1$

Hence , the solution set consists of **a0=0.5211 , a1=1.1351 ,**

Hence the fitting least square line

$$Y = a0e^{a1X}$$

$$Y = 0.5211e^{1.135\,X}$$

$$S_r = \sum_{i=1}^{n}(yi - f(xi))^2$$

i	X	Y	S$_r$
1	2	5.1	0.0031
2	2.3	7.2	0.0121
3	2.6	9.5	0.2172
4	2.9	14.1	0.0083
5	3.2	20	0.0952
		55.9	0.3360

S$_r$ =0.336

$$S_t = \sum_{i=1}^{n}(yi - y)^2$$

$$X = \frac{13}{5} = 2.6 \qquad y = \frac{55.9}{5} = 11.18$$

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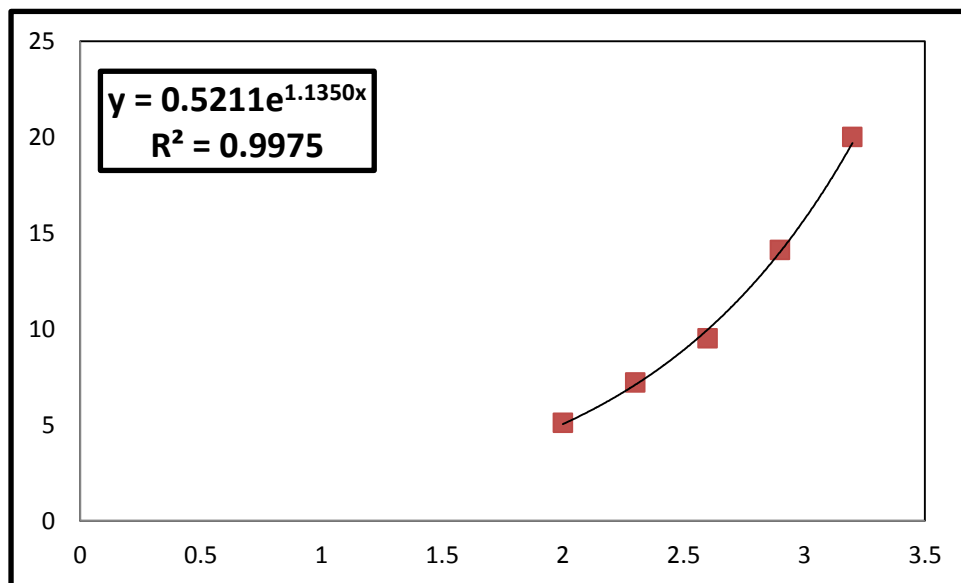
i	y	S_t
1	5.1	36.9664
2	7.2	15.8404
3	9.5	2.8224
4	14.1	8.5264
5	20	77.7924
$\sum$		**141.9480**

S_t = 141.948

$$r = \sqrt{\frac{st-sr}{st}} = \sqrt{\frac{141.948-0.336}{141.948}} = \mathbf{0.9988}$$

r^2 = 0.9976 , *so* **99.76** *of the original data are represented by our linear model.*

- Graph New the equation **Ln(y) = Ln(a_0) + a_1X**

y = 0.5211e^{1.1350x}
R² = 0.9975

Problem 6

Given

x	-3	-1	1	3
y	15	5	1	5

$$aN + b\sum_{i=1}^{n} xi = \sum_{i=1}^{n} yi$$

$$a\sum_{i=1}^{n} xi + b\sum_{i=1}^{n} xi2 = \sum_{i=1}^{n} xiyi$$

(2)

Required

Fit the following data to the:

- $Y = \dfrac{1}{a0 + a1X}$
- $Y = (a0 + a1X)^{-2}$

Solution :

$$(1/y) = a_0 + a_1 X \qquad Y = A + B X$$

i	X	y	Y=1/y	X2	X*Y
1	-3	15	0.0667	9.00	-0.20
2	-1	5	0.2	1.00	-0.20
3	1	1	1	1.00	1.00
4	3	5	0.20	9.00	0.60
$\sum$	**0.00**		**1.47**	**20.00**	**1.20**

Now, eq (1) becomes .

$$4 A = 1.471$$

$$20 B = 1.2$$

(2)`

- *Direct . in this system is solving*

That Is :

$$A = \frac{1.471}{4} = 0.3667$$

$$B = \frac{1.2}{20} = 0.0599$$

- *Hence , the solution set consists of* **a_0=0.3667   , a_1=0.0599**

Hence the fitting least square line

$$1/Y = a_0 + a_1*X$$

$$1/Y = 0.3667 + 0.0599*X$$

$$S_r = \sum_{i=1}^{n}(yi - f(xi))^2$$

i	X	y	S_r
1	-3	15.0000	0.014
2	-1	5	0.011
3	1	1	0.329
4	3	5	0.120
		26.0000	0.4747

S_r =0.4747

$$S_t = \sum_{i=1}^{n}(yi - y)^2$$

$$X=0 \qquad y=\frac{1.4667}{4}=0.3667$$

i	y	S_t
1	15.0000	0.0900
2	5	0.0278
3	1	0.4011
4	5	0.0278
$\sum$		0.5467

S_t = 0.5467

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$$r = \sqrt{\frac{st-sr}{st}} = \sqrt{\frac{0.5467-0.4747}{0.5467}} = \mathbf{0.1317}$$

r^2 = **0.1317** , *so* **13.17** *of the original data are represented by our linear model.*

- Graph New the equation **$1/Y = a_0 + a_1X$**

$y = 1/(0.06x + 0.3667)$
$R^2 = 0.1317$
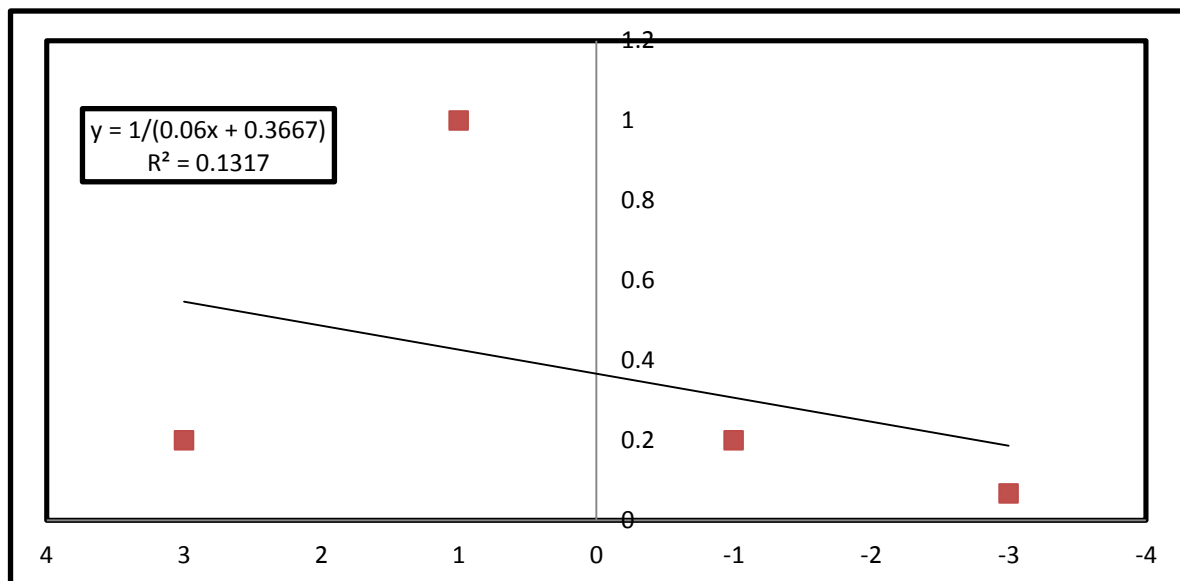
Problem 6 b

$$Y=\sqrt{\frac{1}{y}}$$

i	X	y	Y	X2	X*Y
1	-3	15	0.2582	9.00	-0.77
2	-1	5	0.4472	1.00	-0.45
3	1	1	1.0000	1.00	1.00
4	3	5	0.4472	9.00	1.34
$\sum$	**0.00**		**2.1526**	**20.00**	**1.1198**

Now, eq (1) becomes .

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$$4\ A\ = 2.1526$$

$$20\ B = 1.1198$$ (2)``

in this system is solving Direct .

That Is :

$$A = \frac{2.1526}{4} = 0.5382$$

$$B = \frac{1.1198}{20} = 0.05599$$

Hence , the solution set consists of **a0=0.5382 , a1=0.05599**

Hence the fitting least square line

$$\sqrt{\frac{1}{y}} = a_0 + a_1\ x$$

$$Y = A + B*X$$

$$Y = 0.5382 + 0.05599*X$$

$$S_r = \sum_{i=1}^{n}(yi - f(xi))^2$$

i	X	Y	S_r
1	-3	15.0000	0.0101
2	-1	5	0.0010
3	1	1	0.1615
4	3	5	0.0733
			0.2458

$S_r = 0.2458$

$$S_t = \sum_{i=1}^{n}(yi - y)^2$$

$$X=0 \quad y = \frac{(2.1526)}{4} = 0.5382$$

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i	y	S_t
1	15.0000	0.0784
2	5	0.0083
3	1	0.2133
4	5	0.0083
$\sum$		**0.3082**

$S_t = 0.3082$

$$r = \sqrt{\frac{st - sr}{st}} = \sqrt{\frac{0.3082 - 0.2458}{0.3082}} = 0.4500$$

$r^2 = 0.2025$, *so* **20.25** *of the original data are represented by our linear model*

- Graph New the equation $(1/Y)^{0.5} = a_0 + a_1 X$

$y = (0.056x + 0.5382)^{-2}$
$R^2 = 0.2034$
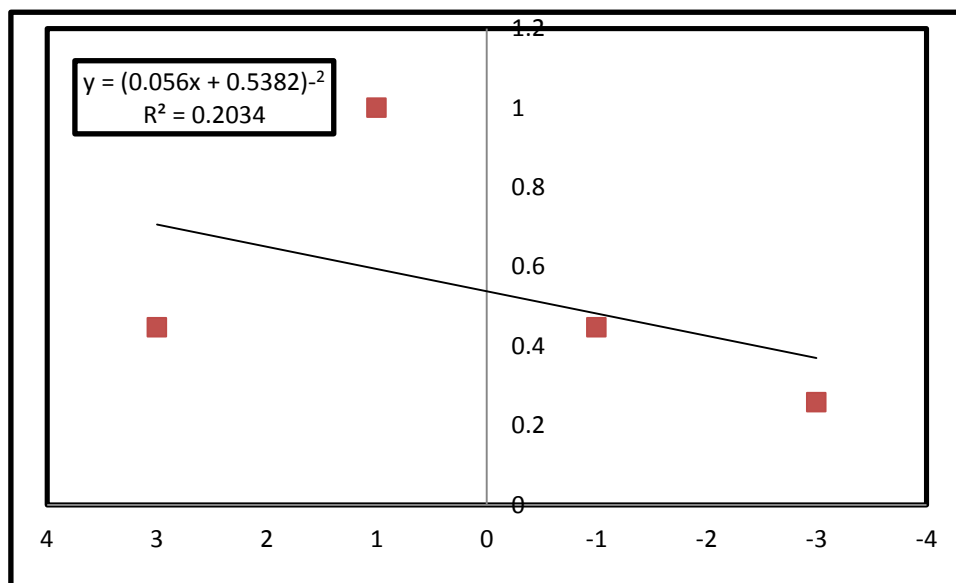
x	42	37	46	30	50	43	46	46
y	173	149	185	123	201	174	188	198

$$aN \; + \; b\sum_{i=1}^{n} xi = \sum_{i=1}^{n} yi$$

$$a\sum_{i=1}^{n} xi + b\sum_{i=1}^{n} xi2 = \sum_{i=1}^{n} xiyi$$

(1)

**Required**

Fit the following data to the:

- _a-exponential function_ $Y = a_0 X^{a1}$
- _**b**- $y = a_0 + a_1 X + a_2 X$_

**Solution** :

i	X	y	Y=ln(y)	X= ln(x)	X2	X*Y
1	42	173	5.1533	3.7377	13.9702	19.2613
2	37	149	5.0039	3.6109	13.0387	18.0688
3	46	185	5.2204	3.8286	14.6585	19.9869
4	30	123	4.8122	3.4012	11.5681	16.3672
5	50	201	5.3033	3.9120	15.3039	20.7467
6	43	174	5.1591	3.7612	14.1466	19.4042
7	46	188	5.2364	3.8286	14.6585	20.0485
8	46	198	5.2883	3.8286	14.6585	20.2469
$\sum$			41.1768	29.9089	112.0031	154.1304

Now, eq (1) becomes .

$$8A \; + \; 29.9089 \; B \; = 41.1768$$

(1)`

$$29.9089 \; A \; + 112.003 \; B = 154.1304$$

in this system is solving by using guess elimination

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$$\begin{pmatrix} 8 & 29.9089 \\ 29.9089 & 112.003 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 41.1768 \\ 154.1304 \end{pmatrix}$$

R1= R1/8

$$\begin{pmatrix} 1 & 3.7386 \\ 29.9089 & 112.003 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 5.1471 \\ 154.1304 \end{pmatrix}$$

R2= R2-29.9089*R1

$$\begin{pmatrix} 1 & 3.7386 \\ 0 & 0.1857 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 5.1471 \\ 0.1863 \end{pmatrix}$$

R2= R2/0.1857

$$\begin{pmatrix} 1 & 3.7386 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 5.1471 \\ 1.0032 \end{pmatrix}$$

Now, this augmented matrix represents the equivalent linear system.

$1 A + 3.7386 B = 5.1471$ (2)

$B = 1.0032$ (3)

Since B = 1.0032 from the last equation ,substituting in the equation (2) by B

A + 3.7386 B =5.1471

A + 3.7386(1.0032)=5.1471

That is,   *A=1.3964*

but A = ln(a_0) ,a_0= e^A , B = a_1

Hence , the solution set consists of **a_0=4.0406 , a_1=1.0032**

Hence the fitting

$Y = a_0 X^{a1}$

$Y = 4.0406 X^{1.0032}$

$$S_r = \sum_{i=1}^{n} (yi - f(xi))^2$$

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i	X	Y=ln(y)	S_r
1	42	173	1.5696
2	37	149	5.0163
3	46	185	9.9780
4	30	123	0.2075
5	50	201	12.7807
6	43	174	3.4211
7	46	188	0.0252
8	46	198	96.8494
$\sum$		1391	129.8476

S_r =**129.8476**

$$S_t = \sum_{i=1}^{n}(yi - y)^2$$

$$y = \frac{1391}{8} = 173.875$$

i	Y=ln(y)	S_r
1	173	0.7656
2	149	618.7656
3	185	123.7656
4	123	2588.2656
5	201	735.7656
6	174	0.0156
7	188	199.5156
8	198	582.0156
$\sum$	1391	**4848.8750**

S_t = **4848.875**

$$r = \sqrt{\frac{st-sr}{st}} = \sqrt{\frac{4848.875-129.8476}{4848.875}} = \mathbf{0.9865}$$

r^2 = 0. 9732 , *so* 97.32 *of the original data are represented by our linear model.*

Graph New the equation **Y = 4.0406 X$^{1.0032}$**

$y = 4.0406x^{1.0032}$
$R^2 = 0.9865$
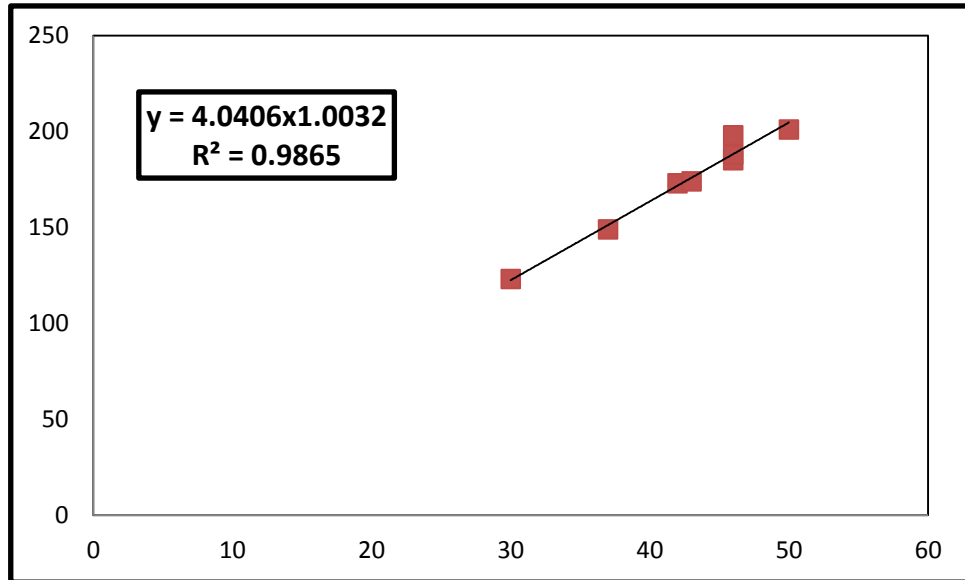
Problem 7b

$$\Sigma x2y = C\ \Sigma x2 + B\ \Sigma x3 + A\ \Sigma x4 \ldots(1$$

$$\Sigma xy = C\ \Sigma x + B\ \Sigma x2 + A\ \Sigma x3 \ldots(2$$

$$\Sigma y = n\ C + B\ \Sigma x + A\ \Sigma x2 \ldots(3$$

i	x	y	x^2	x^3	x^4	x*y	x^2y
1	42	173	1764	74088	3111696	7266	305172
2	37	149	1369	50653	1874161	5513	203981
3	46	185	2116	97336	4477456	8510	391460
4	30	123	900	27000	810000	3690	110700
5	50	201	2500	125000	6250000	10050	502500
6	43	174	1849	79507	3418801	7482	321726
7	46	188	2116	97336	4477456	8648	397808
8	46	198	2116	97336	4477456	9108	418968
Σ	340	1391	14730	648256	28897026	60267	2652315

Putting all these values in (1), (2) and (3), we get

$$2652315 = C\ 14730 + B\ 648256 + A\ 28897026 \ldots(1$$

$$60267 = C\ 340 + B\ 14730 + A\ 648256 \ldots(2$$

1391 = 8C + B 340 + A 14730 ...(3

in this system is solving by using guess elimination

$$\begin{pmatrix} 28837026 & 648256 & 14730 \\ 648256 & 14730 & 340 \\ 14730 & 340 & 8 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 2652315 \\ 60267 \\ 1391 \end{pmatrix}$$

R1 =R1/28837026

$$\begin{pmatrix} 1 & 0.0224 & 0.0005 \\ 340 & 14730 & 340 \\ 14730 & 340 & 8 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 0.0918 \\ 60267 \\ 1391 \end{pmatrix}$$

R2 = R2-340*R1

R3 = R3 -14730*R1

$$\begin{pmatrix} 1 & 0.0224 & 0.0005 \\ 0 & 187.4709 & 9.5573 \\ 0 & 9.5573 & 0.4915 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 0.0918 \\ 766.7901 \\ 39.0062 \end{pmatrix}$$

R2 = R2/187.4709

$$\begin{pmatrix} 1 & 0.0224 & 0.0005 \\ 0 & 1 & 0.051 \\ 0 & 9.5573 & 0.4915 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 0.0918 \\ 4.0902 \\ 39.0062 \end{pmatrix}$$

R3 = R3 – 9.5573*R2

$$\begin{pmatrix} 1 & 0.0224 & 0.0005 \\ 0 & 1 & 0.051 \\ 0 & 0 & 0.0043 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 0.0918 \\ 4.0902 \\ -0.0849 \end{pmatrix}$$

R3 = R3/0.0043

$$\begin{pmatrix} 1 & 0.0224 & 0.0005 \\ 0 & 1 & 0.051 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 0.0918 \\ 4.0902 \\ -19.848 \end{pmatrix}$$

Now, this augmented matrix represents the equivalent linear system.

A + 0.0224 B + 0.0005 C = -0.0918   (1)

$$B + \ 0.051 \ C \ = \ 4.0902 \qquad (2)$$
$$C = -19.848 \qquad (3)$$

Since **C** = -19.848 from the last equation ,substituting in the equation (2) by **A**

$$B + 0.051 \ C = 4.0902$$

$$B + 0.051(-19.848)=4.0902$$

That is, $\qquad\qquad\qquad$ *B =5.102*

Substituting by **C ,B** in the equation (1).

$$A + 0.0224B + 0.0005C= -0.0918$$

$$A +0.0224(5.102) + 0.0005(-19.848)=-0.0918$$

That is , $\qquad\qquad\qquad$ *A= -0.0126.*

Hence , the solution set consists of **A=- 0.0126** , B=*5.102* , C =-19.848

Hence the fitting least square line
$$Y= -0.0126 \ X^2 +5.102 \ X-19.848$$

$$S_r =\sum_{i=1}^{n}\left(yi - f(xi)\right)^2$$

I	X	Y	Sr
1	42	173	0.6247
2	37	149	7.1642
3	46	185	10.1277
4	30	123	1.2724
5	50	201	7.5735
6	43	174	5.0203
7	46	188	0.0333
8	46	198	96.3853
Σ		**1391**	**128.201**

S_R =128.201

$$S_t = \sum_{i=1}^{n}(yi - y)^2$$

$$y = \frac{1391}{8} = 173.875$$

I	Y	ST
1	173	0.765625
2	149	618.7656
3	185	123.7656
4	123	2588.266
5	201	735.7656
6	174	0.015625
7	188	199.5156
8	198	582.0156
Σ		**4848.88**

$$r = \sqrt{\frac{st-sr}{st}} = \sqrt{\frac{4848.88-128.201}{4848.88}} = 0.9867$$

$r^2 = 0.9736$, *so* **97.36** *of the original data are represented by our linear model.*

Graph New the equation **Y= -0.0126 X^2 +5.102 X-19.848**

y = -0.0126x^2 + 5.102x - 19.848
R^2 = 0.9736
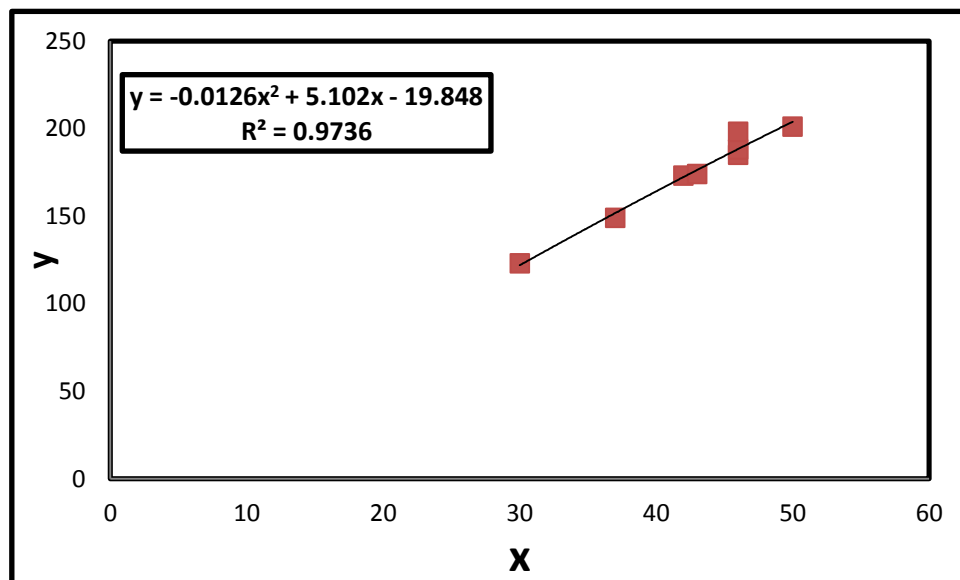
Problem 11

x	1	1.5	3.5	5	6.5
y	1.23	1.67	2.61	2.76	2.66

$$aN + b\sum_{i=1}^{n} xi = \sum_{i=1}^{n} yi$$

$$a\sum_{i=1}^{n} xi + b\sum_{i=1}^{n} xi2 = \sum_{i=1}^{n} xiyi$$

(2)

Required

Fit the following data to the:

$$Y = a \, Xe^{bX}$$

Solution :

i	x	y	y/x	ln(y/x)	x2	x*y
1	1	1.23	1.23	0.2070	1	0.2070
2	1.5	1.67	1.1133	0.1074	2.25	0.1610
3	3.5	2.61	0.7457	-0.2934	12.25	-1.0269
4	5	2.76	0.5520	-0.5942	25	-2.9710
5	6.5	2.66	0.4092	-0.8935	42.25	-5.8076
$\sum$	17.5			-1.4667	82.75	-9.4375

Now, eq (1) becomes .

$$5 A + 17.5 B = -1.4667$$

(1)`

$$17.5 A + 82.75 B = -9.4375$$

in this system is solving by using guess elimination

$$\begin{pmatrix} 5 & 17.5 \\ 17.5 & 82.75 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -1.4667 \\ -9.4375 \end{pmatrix}$$

R1= R1/5

$$\begin{pmatrix} 1 & 3.5 \\ 17.5 & 82.75 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -0.2933 \\ -9.4375 \end{pmatrix}$$

R2= R2-17.5R1

$$\begin{pmatrix} 1 & 3.5 \\ 0 & 21.5 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -0.2933 \\ -4.3048 \end{pmatrix}$$

R2= R2/0.9

$$\begin{pmatrix} 1 & 3.5 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -0.2933 \\ -0.2002 \end{pmatrix}$$

Now, this augmented matrix represents the equivalent linear system.

1 A + 3.5 B = -0.2933 (2)

B = -0.2002 (3)

Since B = -0.2002 from the last equation ,substituting in the equation (2) by B

A + 3.5 B=-0.2933

A + 3.5(-0.2002)=-0.2933

That is, *A=0.4073*

but A = ln(a) ,a= e^A , B = b

Hence , the solution set consists of **a=1.5028 , b=-0.2002 ,**

Hence the fitting least square line

$$Y = a\, X e^{bX}$$
$$Y = 1.5028\, X e^{-0.2002\, X}$$

$$S_r = \sum_{i=1}^{n}(yi - f(xi))^2$$

i	X	Y=ln(y/x)	Sr
1	1	1.23	0.0000
2	1.5	1.67	0.0000
3	3.5	2.61	0.0000
4	5	2.76	0.0000
5	6.5	2.66	0.0000
$\sum$		10.9300	0.0000

$S_r = 0$

$y = 1.5028e^{-0.2002x}$
$R^2 = 1.0000$